To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count

Pierina Cheung[1,3], Miriam Rubenson[2,3], and David Barner[3,4]

[1]Department of Psychology, Wesleyan University

[2]Department of Psychology, University of Southern California

[3]Department of Psychology, University of California, San Diego

[4]Department of Linguistics, University of California, San Diego

Word Count: 8877 words

Corresponding Author:

Pierina Cheung

Department of Psychology, Wesleyan University

207 High Street, Middletown, CT 06457

cheung.pierina@gmail.com

Abstract

Recent accounts of number word learning posit that when children learn to accurately count sets (i.e., become "cardinal principle" or "CP" knowers), they have a conceptual insight about how the count list implements the successor function – i.e., that every natural number *n* has a successor defined as *n*+1 (Carey, 2004, 2009; Sarnecka & Carey, 2008). However, recent studies suggest that knowledge of the successor function emerges sometime after children learn to accurately count, though it remains unknown when this occurs, and what causes this developmental transition. We tested knowledge of the successor function in 100 children aged 4 through 7 and asked how age and counting ability are related to: (1) children's ability to infer the successors of all numbers in their count list, and (2) knowledge that *all* numbers have a successor. We found that children do not acquire these two facets of the successor function until they are about 5½ or 6 years of age – roughly 2 years after they learn to accurately count sets and become CP-knowers. These findings show that acquisition of the successor function is highly protracted, providing the strongest evidence yet that it cannot drive the cardinal principle induction. We suggest that counting experience, as well as knowledge of recursive counting structures, may instead drive the learning of the successor function.

(217 words)

Keywords: cardinal principle; successor function; infinity; count list; conceptual change; natural number concepts

As adults, we understand that the natural numbers are infinite, and thus that for any number, no matter how large, a larger number can be created by adding 'one'. This simple principle – sometimes called the "successor function" – lies at the foundation of human arithmetic understanding, and by most current accounts is learned by children in the US by around the age of 4 (Carey, 2004, 2009; Sarnecka & Carey, 2008). According to these accounts, children acquire the successor function via an inductive inference over the words *one, two,* and *three*, an inference that leads them to understand the cardinal principle – i.e., the last word in a count labels the cardinality of the counted set (Carey, 2004, 2009; Sarnecka & Carey, 2008; Sarnecka, 2015; Schaeffer, Eggleston, & Scott, 1974). Thus, on this view, learning to count is not simply a matter of memorizing a list of numbers and attaching it to a counting procedure, but instead involves an important conceptual change, one that involves understanding the meaning of the order of number words. Other studies, however, have found that the successor function emerges sometime after children become competent counters. None of these studies has evaluated when knowledge of the successor function becomes productive (i.e., true of all possible numbers), or how children might make such an inductive inference. In the present study, we tested children's knowledge of the unbounded nature of the successor function, and provide evidence that productive knowledge of the successor function is in fact acquired much later than previously thought – as late as 6 years of age.

In the US, middle class English-speaking children typically begin number word learning sometime around the age of 2 when they memorize a subset of the count list (e.g., *one, two, three, four, five, six,* etc.), and begin to recite it much like they do the alphabet (for details, see Briars & Siegler, 1984; Fuson, 1988; Fuson & Hall, 1983; Gelman & Gallistel, 1978). However, initially children have little idea of what these count routines mean, such that when asked for a small set of objects – e.g., even one or two – they give a random amount (e.g., Schaeffer et al., 1974). Children learn their first few number word meanings slowly and in sequence, beginning with *one*, at which point they can be called "one-knowers". When tested with Wynn's (1990, 1992) Give-a-Number task, these children give one object when asked for one, but generally do not give one for higher numbers. Several months later children acquire the meaning of *two,* at which point they are known as "two-knowers", and give two objects when asked for two, but not when asked for three, four, etc. Next they learn *three* ("three-knowers") and then *four* ("four-knowers") in sequence over several additional months. Together, these groups of children are known as "subset-knowers", since they know the meanings of only a subset of their number words. Finally, sometime between the ages of 3 and 4, most children become "cardinal principle knowers" or CP-knowers, and can use their counting routine to label and generate any sets within their counting range (for discussion of this developmental trajectory, see Sarnecka & Lee, 2009). Although the precise timing of these stages varies across populations, the basic sequence has been reported across a variety of distinct linguistic and cultural groups including children in Canada, Japan, Taiwan, China, Russia, Bolivia, Slovenia, and Saudi Arabia (see Almoammer, Sullivan, Donlan, Marušič, Žaucer, O'Donnell, & Barner, D., 2013; Barner, Chow, & Yang, 2009; Barner, Libenson, Cheung, & Takasaki, 2009; Le Corre, Van de Walle, Brannon, & Carey, 2006; Le Corre, Li, Huang, Jia, & Carey, in press; Piantadosi, Jara-Ettinger, & Gibson, 2014; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007).

According to some accounts (e.g., Carey, 2009), the transition to becoming a CP-knower marks a profound inductive leap. After having constructed meanings for *one, two,* and *three* over many months without noticing their relationship to counting, children identify an isomorphism between the sets denoted by these words and order in the count list. Specifically, children notice that for any number *n*, the meaning of that number's successor in the count list is exactly *n+1* (Sarnecka & Carey, 2008). Critically, according to this account, this knowledge, which "turns a subset knower into a cardinal-principle knower" is thought to reflect implicit knowledge of the successor function (p. 673), as described by the Peano-Dedekind axioms, which in turn provide a logical foundation for natural number. Below, a relevant subset of these axioms is described,[1] including the successor function and its induction in (2) and (3):

(1) 1 is a natural number.

(2) For every natural number *n*, the successor of *n* is a natural number.

(3) Every natural number has a successor.

Thus, contrary to earlier formulations (Gelman & Gallistel, 1978; see also Leslie, Gallistel, & Gelman, 2007), this view holds that the logic of counting is not innate, but instead is constructed inductively over representations of small numbers, which are themselves constructed from representations of objects and sets (for an early formulation of this same idea, see Mill, 1843, and Frege, 1884, for discussion; see also Hurford, 1987).[2]

However, the hypothesis that children become CP-knowers by acquiring the successor function has recently come under scrutiny. One problem, first noted by Sarnecka and Carey (2008), is that although the transition from subset knower to CP-knower can be described as a

---

[1] Omitted are principles which define exact equality, rule out zero as a successor to a number, and which define the successor function as an injection (i.e., preserving one-to-one correspondence).

[2] According to Mill, "we may call "Three is two and one" a definition of three; but the calculations which depend on that proposition do not follow from the definition itself, but from an arithmetical theorem presupposed in it, namely, that collections of objects exist, which while they impress the senses thus, ∴ , may be separated into two parts, thus, ... This proposition being granted, we term all such parcels Threes, after which the enunciation of the above-mentioned physical fact will serve also for a definition of the word Three." (p. 191)

conceptual change rooted in the discovery of the successor function, it can also be described more modestly as a process of discovering a procedure for labeling and generating sets:

> "The cardinal principle is often informally described as stating that the last numeral used in counting tells how many things are in the whole set. If we interpret this literally, then the cardinal principle is a procedural rule about counting and answering the question 'how many'… Alternatively, the cardinal principle can be viewed as something more profound – a principle stating that a numeral's cardinal meaning is determined by its ordinal position in the list… If so, then knowing the cardinal principle means having some implicit knowledge of the successor function – some understanding that the cardinality for each numeral is generated by adding one to the cardinality for the previous numeral." (p. 664).

Consistent with the procedural learning hypothesis, Sarnecka and Carey note that several points in children's developmental trajectories once thought to involve conceptual understanding were later shown to actually rely on relatively unanalyzed routines. For example, as shown by several early studies (Fuson, 1988; Fuson, Pergament, Lyons, & Hall, 1985; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Schaeffer, et al., 1974), when relatively young children (e.g., 3-year-olds) are asked "how many" objects are in a set, they are often able to count a set, ending with the correct numeral. However, when these same children are then asked again how many objects there are, they either respond with an unrelated number, or simply begin counting again. Furthermore, many children who *can* respond correctly to this question are subsequently incapable of using their count to give the experimenter the requested amount (e.g., Schaeffer et al., 1974), and are not CP-knowers according to standard methods (like Wynn's Give-a-Number task) because they fail to count accurately (e.g., Fuson et al., 1985). It is thus possible that becoming a CP-knower, like learning of the last-word rule, is yet another instance of procedural learning – i.e., in response to a request for *n* objects, one should count to *n* and give all objects implicated in the count.

Against the procedural learning hypothesis, Sarnecka and Carey presented evidence that CP-knowers, as a group, understand something more than just a procedure when they count. Specifically, they found that CP-knowers, but not subset knowers, know that adding one object to a set corresponds to an increase of exactly one in the count list. To show this, Sarnecka and Carey presented children with a task we will call the "Successor Task", in which an experimenter placed a set of objects into a container and labeled the set with a number larger than three (e.g., 5), and then added either one or two objects to the container. Children were then asked how many objects were in the container in a forced-choice. For example, if 5 objects were originally hidden and 1 object was added, they were asked whether there were now 6 objects or 7 objects in the container. Overall, Sarnecka and Carey found that CP-knowers provided correct responses to these questions 66% of the time, whereas one-, two-, and three-knowers performed at or near chance levels of 50%. On the basis of this, they argued that discovering the successor function is what causes children to become CP-knowers.

There are two important problems with this conclusion. The first problem is that, although CP-knowers in Sarnecka and Carey's study performed better on average than subset knowers, many CP-knowers nevertheless appeared to lack knowledge of the successor function. Specifically, out of 29 CP-knowers in the study, 13 performed at or below chance on the task, while only 5 performed at ceiling. Such a result is surprising if mastery of the knowledge measured by this task – i.e., that the successor of *n* is $n+1$ – is what *causes* children to become CP-knowers. Instead, the results appear to indicate the opposite: That children become CP-knowers before they learn the successor function. In support of this, a subsequent study by Davidson, Eng, and Barner (2012) replicated this pattern in CP-knowers, but also showed that (1) children's performance on Sarnecka and Carey's Successor Task was predicted by how high children could count, (2) children's ability to identify a successor differed according to the size of the number tested, even for numbers well within children's counts lists, and (3) only the most skilled counters systematically gave correct responses for even very small numbers like 4 and 5, and no children succeeded systematically for numbers higher than 10. Thus, this study suggested that many CP-knowers do not understand the successor function at all, and also that those who do show evidence of understanding the function only do so for the very smallest numbers. Against what would be predicted by an inductive inference, CP-knowers fail to generalize the principle to their entire count list (see also Wagner, Kimura, Cheung, & Barner, 2015, for similar findings).

The second problem with Sarnecka and Carey's evidence is that the Successor Task does not probe knowledge of the successor function as it is typically defined. According to the Peano axioms, the successor function states that *every* natural number has a unique successor. This formulation is meaningfully stronger than the one often used in the developmental literature, which states only that "each number is generated by adding one to the previous number" (Sarnecka & Carey, 2008; Sarnecka, 2015). In particular, the full-fledged successor function implies that there is no largest number – a critical foundation to understanding natural number. A child who truly understands the successor function should not only know that the successor of 5 is 6, or that the successor of 16 is 17, but also that for *any* number, n, this number has a unique successor whose value is $n+1$, and thus, there is no highest number.

Several past studies have explored children's intuitive understanding of infinity and have found that it emerges relatively late. Although even adults struggle with the formal notion of infinity (e.g., Wheeler & Martin, 1988), most numerate adults know that "one" can be recursively added to any number, and children appear to learn this sometime around the age of 6 (with 7- to 8-year-olds speaking explicitly of infinity). For example, Gelman and colleagues asked 5- to 9-year-old children questions about iterative addition and the existence of a biggest number in qualitative interviews (Evans, 1983; Gelman, 1980; Hartnett, 1991; Hartnett & Gelman, 1998). They found that children's understanding of infinity develops in three stages. Children in the first stage claim that there is a biggest number and that one cannot add one to it. This is followed by a stage in which children provide somewhat paradoxical answers: They tend to respond that one can keep adding but at the same time think that there is an end to numbers. At the last stage, children typically respond immediately that there is no biggest number and discuss, without probing, the infinite nature of numbers. These findings suggest that the development of infinity understanding is protracted, raising the question about how it may be related to children's acquisition of the successor function.

Currently, it is unknown when children acquire knowledge that all numbers have a successor. Previous studies have failed to test children older than 5, and have found that children below this age have only item-based knowledge of particular successor relations, rather than a general principle that governs all possible numbers – or even all numbers within their productive count lists. Further, past studies have assessed children using the Successor Task, which cannot alone identify knowledge of the successor function, since it asks only about particular numbers. To remedy these issues, we first tested children between the ages of 4 and 7 using Sarnecka and Carey's Successor Task to probe when children can reason about successor relations for known numbers. Although this task cannot alone demonstrate knowledge of the successor function, it can identify children who lack such knowledge: Evidence that children can only identify particular successor relations for a subset of the numerals in their count list would suggest that they only have item-based knowledge of the successor function. Second, we supplemented this task with a battery of questions adapted from Gelman and colleagues, which probed when children understand that *every* number has a successor, and thus that numbers are infinite. By combining data across tasks, we identified children who likely understood the successor function as defined in the Peano axioms – i.e., that every number n has a successor n+1, such that numbers never end.

Method

Participants

One hundred English-speaking children aged four to six years (M = 5;6; range = 4;0 – 6;11) participated in the study.[3] They were recruited at museums and childcare centers in the San Diego metropolitan area. An additional 24 children participated but were excluded from the analysis due to a failure to show knowledge of the cardinal principle (n = 20), having a primary language other than English (n = 1), parental interference (n = 1), or experimenter error (n = 2).

Procedures

Children were invited to sit across from the experimenter to play a game. The testing session, which lasted approximately 15 minutes, consisted of four tasks: A “Give-a-Number” task to identify Cardinal Principle knowers; a Highest Count task to assess children’s exposure to number words; a Successor Task to test children’s knowledge of particular successor relations;

[3] This sample size was chosen prior to the beginning of the study based on a power analysis which estimated that 32 children for each age group would provide sufficient power to detect a small to medium effect with alpha = .80 using three predictors in a linear regression.

and an Infinity Task – a set of open-ended questions related to the concept of infinity, which tested whether children thought all possible numbers have a successor. All children first completed the Give-a-Number task, followed by the Highest Count task. Children identified as CP-knowers then completed the Successor Task and the Infinity Task, the order of which was counterbalanced across children.

Give-A-Number task. This task was adapted from Wynn (1990) and was used to assess children's comprehension of number words. The experimenter began by presenting the child with 12 small plastic bears and inviting the child to play a game with her toys. For each trial, the experimenter asked the child, "Can you give me *N* bears?" Once the child responded, the experimenter then asked, "Is that *N*? Can you count and make sure?" and encouraged the child to count the bears. If the child recognized an error, the experimenter allowed the child to change his or her response.

Children were asked to give seven and eight bears twice (7, 8, 7, 8). Children who succeeded on all four trials were deemed CP-knowers and were asked to complete the subsequent tasks in the experiment. Children who did not succeed on all four trials were thanked for their participation and given a small prize.

Highest Count task. Next the experimenter asked each child, "Can you count as high as you can for me?" If the child failed to respond, the experimenter said, "Let's count together! One…" with rising intonation in an attempt to encourage the child to continue counting alone. The experimenter stopped the task whenever a child successfully counted up to 100.
The experimenter recorded the numbers recited, noting any errors such as omission (e.g., "…11, 12, 14") and cyclical repetition (e.g., "…28, 29, 30, 21, 22"). The child's highest count was

defined as the largest number counted to before an error. For example, nineteen was the highest number recorded for a child who omitted twenty.

Successor Task. This task was modeled after Sarnecka and Carey (2008) and was designed to assess children's understanding of the successor function—more specifically that adding one object to a set (i.e., *N*) results in an increase of exactly one unit on the count list (i.e., $N + 1$). The experimenter presented children with an opaque box and a small blue container filled with identical plastic bears. To begin each trial, the experimenter directed the child's attention to the box by exclaiming, "Look, there's nothing in the box!" and permitted the child to look inside to confirm that the box was empty. The experimenter then held up the container of objects, stating, "I have *N* bears. I'm putting *N* bears in the box," where *N* equaled the number of objects tested on that trial. She then poured the *N* bears into the box and closed the lid to prevent the child from using perceptual cues to answer questions regarding quantity.

To ensure that the child remembered the number of items in the box, the experimenter asked, "How many bears are in the box?" If the child answered incorrectly or failed to respond, the experimenter said, "Oops, let's try again!" Objects were removed and the trial was repeated until the child responded correctly to the memory-check question.

Next, the child was told, "Right! Now watch," as the experimenter added one bear to the box. She then elicited a choice between $N + 1$ and $N + 2$ by saying, for instance, "Are there thirteen bears or fourteen bears in the box?" Irrespective of their response, children received neutral feedback (e.g., "Thank you") and the next trial began. Children were tested on a wider range of numbers than in previous studies (Sarnecka & Carey, 2008; Davidson et al., 2012; Wagner et al., in press). Specifically, numbers were divided into four ranges: Small (4, 5, 7), Medium (12, 16, 18), Large (23, 24, 28, 31, 35, 36), and Very Large (53, 57, 76, 77) numbers.

The order in which the choice alternatives (i.e., $N + 1$ or $N + 2$) were presented was counterbalanced across trials and presentation of the trials was randomized for each child. Children were not tested on numbers that exceeded the highest number they could count to, and thus received different number of trials depending on their highest count.

Infinity Task. To assess children's understanding of infinity, the experimenter asked a series of open-ended questions based on previous studies by Evans (1983) and Hartnett and Gelman (1998). All children were asked six questions, in a fixed order. The first three questions asked whether children thought there was a highest number:

(1) "What is the biggest number you can think about?" If the child did not answer, the experimenter probed them by asking how high they could count. A majority of children provided a largest number without this probe.

(2) They were next asked, "Is that the biggest number there could ever be?" If the child responded "yes," then the experimenter moved on to the next question. If the response was "no," the experimenter asked, "Can you think of a bigger number?" If the child said a larger number, the experimenter repeated the question, "Is that the biggest number that could ever be?" If the child again answered "No," the experimenter repeated this exchange up to four times, or until the child responded in the affirmative.

(3) Next they were asked, "If I keep counting, will I ever get to the end of numbers, or do numbers go on forever?" Regardless of their choice, each child was asked "Why?"

The final three questions probed children's understanding of the successor function:

(4) "If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn't add any more?" All children were asked "Why?" in response to whichever choice they made.

(5) Next the experimenter asked, "You said that the biggest number you know is *X*. Tell me: Is it possible to add one to *X*, or is *X* the biggest number possible?" For *X*, the experimenter stated the largest number the child had yet used during the testing session, and thus, in some cases, it was different from the child's response to the first question. The child was again asked "Why?" in response to whichever choice they made.

(6) Finally, the child was asked, "Could I keep adding one?" and "Why?" or "Why not?" For the last question, if the child responded in the affirmative, the experimenter asked a seventh question, "What would happen if I kept adding one?"

Coding

Infinity Task. Using criteria established by previous studies of infinity understanding (Evans, 1983; Gelman, 1980; Hartnett & Gelman, 1998), we classified children into four different stages. Specifically, using their responses and justifications for Questions 1-3 we determined whether each child believed that there was a highest number, and using Questions 4-6 we determined whether they believed it was always possible to add 1 to a number. This scheme resulted in four possible categories, depicted in Table 1, similar to Evans (1983) and Hartnett and Gelman (1998). We classified children as: Full Infinity Knowers if they believed both that it was possible to add 1 indefinitely and that there was no highest number; Successor Only Knowers if they believed that it was possible to add 1 indefinitely but that there was nevertheless a highest number; Endless Only Knowers if they believed that it was not possible to add 1 but that there was no highest number; Infinity Non-Knowers if they believed that there was a highest number and that it was not possible to add 1 indefinitely. The coding scheme is presented in Appendix A.

Assignment of children to these four levels was conducted independently by the first and second authors. Initial agreement was 81.0% (Cohen's Kappa = .73, $p < .001$), indicating strong agreement (Landis & Koch, 1977). Disagreements were resolved through additional discussion until consensus was obtained, and before any analyses below were conducted, in order to reduce bias in decision making. Excerpts of transcripts for children in each classification are presented in Appendix B.

Table 1. Four classifications on the knowledge of infinity.

		Can we keep adding?	
		Yes	*No*
Is there a biggest number?	*Yes*	Successor Only Knowers	Infinity Non-Knowers
	No	Full Infinity Knowers	Endless Only Knowers

Results

Successor Task

All children included in the analyses were CP-knowers. Children's highest count ranged from 8 to 100 (*Median* = 40, *Mean* = 55.7, *SD* = 35.1). Consistent with previous reports, children's highest count was significantly correlated with age, $r(98) = .66$, $p < .001$. Table 2 presents children's average age and average highest count. Preliminary results showed no effects of gender or task order (Fs < 1, *ns*) on overall proportion correct on the Successor Task, and thus these variables were collapsed on the Successor Task analyses.

Our first analysis asked whether children's performance on the Successor Task varied as a function of their highest count and the range of numbers tested. If children understand the successor function as a function that applies to all numbers in their count list, then they should be able to infer the value of successors for almost all numbers in their counting range. However, if children have item-based knowledge, they should only be able to identify particular successor relations for some but not all of the numbers in their count list. To address this question, we divided children into different counting groups following previous research (Davidson et al., 2012; Wagner et al., 2015): Low Counters (highest count = 8-19, N = 23), Medium Counters (highest count = 20-39, N = 24), High Counters (highest count = 40-79, N = 19), and Very High Counters (highest count = 80 and above, N = 34). This grouping ensured that small numbers were within the counting range of all Low Counters, small and medium numbers were within the range of all Medium Counters, small, medium, and large numbers were within the range of all High Counters, and lastly, that all numbers tested were within the range of Very High Counters.[4] Thus, we only tested and analyzed data for numbers that were within the limits of children's productive count lists. This approach increased the likelihood of finding early comprehension of the successor function, relative to an approach whereby this knowledge is only accorded to children who can identify all successors for all numbers up to a single standard (e.g., 100), which we judged to be too conservative a threshold.

Table 2. Average age and average highest count by counting group.

	Age		**Highest count**
	Mean (SD)	Range	Mean (SD)
Low Counters	4.82 (.79)	4.02 – 6.89	14.83 (2.95)
Medium Counters	5.10 (.57)	4.01 –6.79	31.50 (5.38)

[4] Note that this grouping ignores the fact that some Low Counters were tested on medium numbers, some Medium Counters were tested on large numbers, and some High Counters were tested on very large numbers. For example, if a child's highest count was 15, they were classified as a Low Counter (defined as having highest count between 8 and 19), and tested on small numbers (4, 5, 7) and a medium number – 12. Given that our primary interest in this analysis was to analyze the effect of count group, we focused on numbers that *all* children in a counting group were tested on.

High Counters	5.58 (.85)	4.21 – 6.89	57.58 (12.81)
Very High Counters	6.25 (.61)	4.71 – 6.95	99.50 (2.12)

Across all counting groups, children performed above chance (.50) for numbers within their counting range: Low Counters (M = 81.2%, SD = 24.3%), $t(22) = 6.16$, $p < .001$; Medium Counters (M = 85.4%, SD = 19.2%), $t(23) = 9.02$, $p < .001$; High Counters (M = 85.0%, SD = 16.1%), $t(18) = 9.47$, $p < .001$; and Very High Counters (M = 96.5%, SD = 5.6%), $t(33) = 48.53$, $p < .001$. Note that this performance was better than that of CP-knowers reported by Sarnecka and Carey (2008) and others (Davidson et al., 2012; Wagner et al., 2015), likely because our CP-knowers were substantially older than children in those studies. Nevertheless, children's ability to identify the successors of numbers differed as a function of both their highest count and the range of numbers tested, indicating a steady developmental change. Figure 1 shows that all children, including Low Counters, performed relatively well on small numbers. However, Medium and High Counters experienced more difficulty with larger numbers (panels B and C). Very High Counters performed at ceiling across their entire counting range (panel D). These findings suggest that many children had item-based knowledge of the successor function – i.e., they could only identify successors for a subset of the numbers within their counting range – while some children (i.e., those with considerable counting experience) could consistently identify successors for all known numbers.

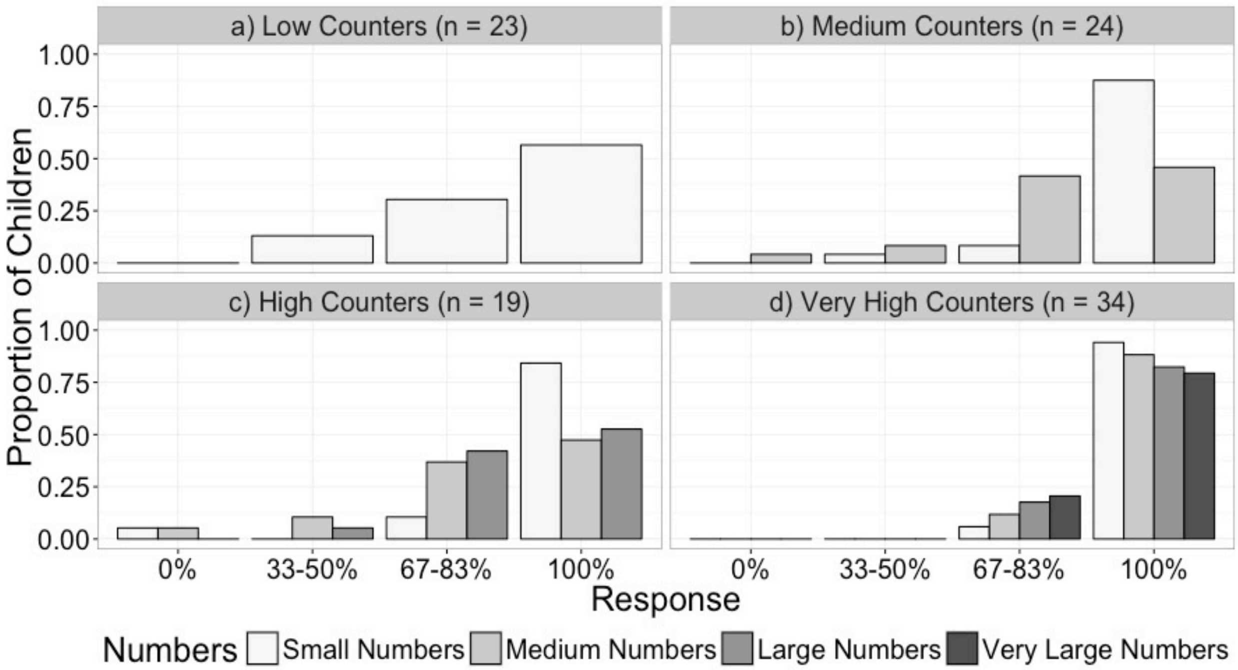

Figure 1. The proportion of Low Counters (panel A), Medium Counters (panel B), High Counters (panel C), and Very High Counters (panel D) who responded correctly between 0-100% on the Successor Task. The different bars represent different number ranges.

We next employed mixed effects models to analyze how children's performance on the Successor Task differed as a function of two continuous variables, Highest Count and Age, and a categorical variable, Number Range (i.e., the range of numbers tested on the Successor Task; Small, Medium, Large, Very Large). We constructed the models in a hierarchy of increasing complexity by first including only the main effects, followed by 2-way interactions and a 3-way interaction, and all models included a random intercept for subjects (lmer function in R; Bates, Maechler, & Bolker, Walker, 2015; R Development Core Team, 2013). Model comparison using the chi-square likelihood ratio test showed that adding the main effects significantly improved the baseline intercept-only model, $\chi^2(5) = 94.72$, $p < .001$, suggesting that overall, children's performance was better for small numbers (Medium Numbers: β = -.16, *SE* = .025; Large Numbers: β = -.08, *SE* = .027; Very Large Numbers: β = -.10, *SE* = .034), when children were older (β = .072, *SE* = .020) and when they could count higher (β = .0019, *SE* = .0005). There was also an interaction between Highest Count and Age, $\chi^2(1) = 11.71$, $p < .001$, indicating that the effect of Age on children's performance was stronger for those whose highest count was lower (see Figure 2). However, adding the Number Range x Age interaction, $\chi^2(3) = 1.03$, $p = .80$, or the 3-way interaction did not significantly improve the fit of the model, $\chi^2(3) = 1.92$, $p = .59$. As predicted, there was a significant interaction between Number Range and Highest Count, $\chi^2(3) = 27.09$, $p < .001$. These results suggest that children performed better for small numbers, and that within each range of numbers, children performed better if they were more competent counters.

To examine how the effects of Number Range differed at each level of children's highest count, separate repeated measures ANOVA were conducted for Medium Counters, High Counters, and Very High Counters, with Number Range as a within-subject variable.[5] This analysis revealed an effect of Number Range for Medium Counters, $F(1,23) = 16.26$, $p < .001$, $\eta_p^2 = .41$, indicating that they performed better on small numbers (*M* = 94.4%, *SD* = 16.1%) than medium numbers (*M* = 76.4%, *SD* = 26.9%). There was no effect of Number Range for High Counters, $F(1.41,25.44) = 2.89$, $p = .089$, or Very High Counters, $F(2.43,80.29) = .93$, $p = .41$. Nevertheless, as shown in Figure 1, the performance of High Counters on the Successor Task was more variable than that of Very High Counters. A Brown-Forsythe test confirmed that the variances between the two groups were not equal, $F = 9.37$, $p = .0035$. Specifically, whereas a high proportion of Very High Counters performed at 100% on the Successor Task across all numbers tested, High Counters performed at ceiling for small numbers, but not for medium and

[5] This analysis was not performed on Low Counters, because only the small numbers were within the range of all Low Counters.

large numbers. Thus, children did not identify the successors for all numbers in their count list until they were Very High Counters and could count to around 80.

These findings indicate that only children with substantial counting experience were able to consistently infer the values of successors for all numbers in their count lists. We should note that this finding is particularly striking given that we only analyzed Successor Task data for numbers within each group's counting range. If anything, this approach should inflate the likelihood that Low Counters reach ceiling on the Successor Task relative to Very High Counters given that the former completed fewer trials and with smaller numbers. The fact that Very High Counters perform better despite this suggests a real difference in how these groups understand successor relations.

To explore how this pattern was related to age, we plotted performance on the Successor Task as a function of Age, separated by Counting Group. Figure 2 shows that performance on the Successor Task increased linearly as a function of Age. After approximately 5½ years of age, there was little variability in children's performance independent of Counting Group, suggesting that they had reached asymptote. The linear trend of Age was present in the Low, Medium, and High Counters, but not in the Very High Counters, who performed above 80% regardless of age (see the last panel of Figure 2). This is consistent with the conclusion that the effect of Age on Successor Task performance was greater for those whose highest count was lower.

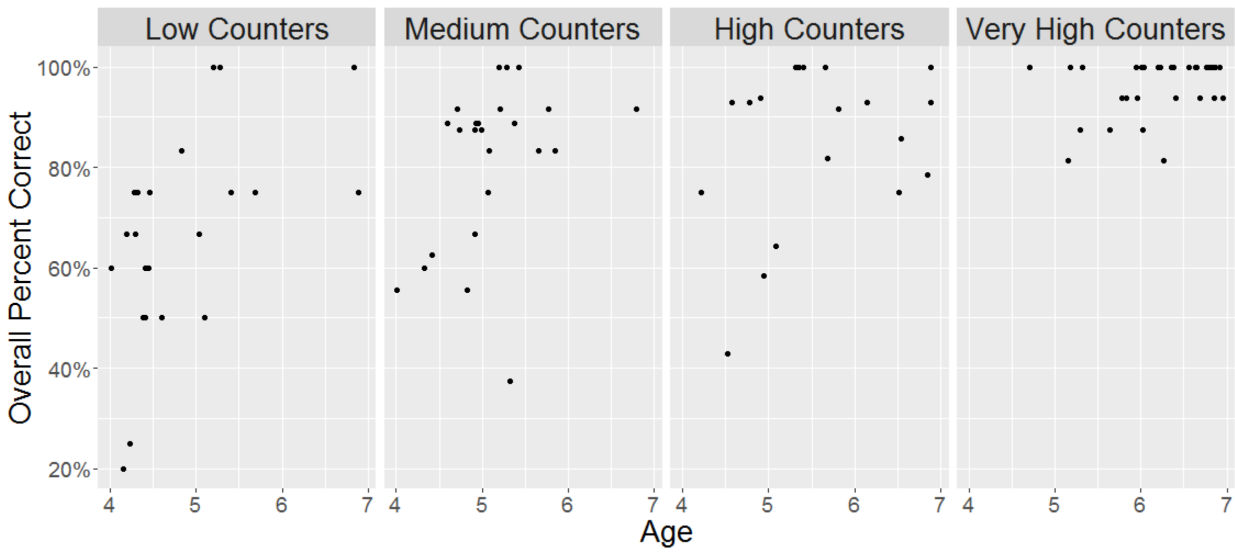

Figure 2. Children's overall performance on the Successor Task by age, separated by counting group.

Infinity Task

Thus far we have shown that until at least 5½ years of age children continue to exhibit predictable developmental variability on the Successor Task. This variability is related to age and children's highest count. However, as noted in the Introduction, the Successor Task alone cannot address when children acquire the full-fledged successor function, since such knowledge involves not only knowledge of particular successor relations, but also of the more general principle that *all* numbers have a unique successor, such that there is no highest number. To explore this, we examined data from the Infinity Task and then tested how these data were related to children's responses on the Successor Task.

Of the 100 children tested, 57 responded that one can always add 1 to a number. Of these, 24 judged that there is no largest number. This group of children is especially interesting, since they appeared to know that every number has a successor, and that numbers are infinite – i.e., knowledge of the successor function as specified by the Peano axioms. We labeled these 24 "Full

Infinity Knowers". The remaining children were assigned to three additional groups. Children who judged that there is a highest number but that one can nevertheless always add 1 were called "Successor Only Knowers". The remaining 43 children judged that one cannot keep adding 1 indefinitely. Of these, 37 judged that there is a largest number. We labeled these children "Infinity Non-Knowers". As in previous studies we found almost no children who judged that there is no largest number but that it is not always possible to add 1. We labeled these children "Endless Only Knowers". Table 3 presents the average age of children in each group.

Before continuing to our main analyses, we should note that whereas Evans (1983) and Hartnett and Gelman (1998) found no Endless Only children, we found 6. Such children may have arisen due to random guessing on the Infinity Task, but another possibility is that some children in this group and in the Full Infinity group were explicitly taught by caregivers that numbers never end – a behavior that may be more likely among caregivers now than in the early 1980's. Consequently, it is possible that the Full Infinity group differs from the Successor Only group purely with respect to explicit coaching about the nature of infinity, and the fact that numbers never end. Against this hypothesis, Endless Only Knowers, although present in our dataset, were nevertheless very infrequent. Also, *post hoc* analyses indicate that children in the Full Infinity group differed from the Successor Only group not only with respect to their belief that numbers never end, but also in every other respect, including their age ($t(55) = -4.35$, $p < .001$, d = 1.17 [0.59, 1.75]; Table 3), their highest count ($t(55) = -2.58$, $p = .013$, d = 0.69 [0.14, 1.25]; Full: $M = 75.2$, $SD = 32.5$; Successor Only: $M = 51.4$, $SD = 32.7$), and their performance on the Successor Task ($t(55) = -2.26$, $p = .028$, d = 0.61 [0.06, 1.16]; Full: $M = 91.5\%$, $SD = 14.7\%$; Successor Only: $M = 80.7\%$, $SD = 19.8\%$). In addition, whereas 19 out of 24 (79.2%) Full Infinity Knowers performed at ceiling (at least 90%) on the Successor Task, only 14 out of

33 (42.4%) of Successor Only Knowers did, $X^2 = 6.26$, $p = .012$, *phi* = .33. Together, these facts suggest that differences in infinity knowledge between these groups are most likely due to global differences in experience with number and counting, and not simply the product of some parents telling their children that numbers never end.

Table 3. Average age and standard deviation for children in each infinity group.

	Age	
	Mean (SD)	Range
Infinity Non-Knowers (N=37)	5.20 (.77)	4.01 – 6.87
Endless Only Knowers (N=6)	5.52 (.74)	4.45 –6.56
Successor Only Knowers (N=33)	5.31 (.91)	4.19 – 6.86
Full Infinity Knowers (N=24)	6.28 (.69)	4.71 – 6.95

Relation between Successor Task and Infinity Task

For the analyses that follow we combined our small sample of Endless Only Knowers with the Successor Only Knowers and labeled this group 'Partial Knowers' (N=39). Overall, the best predictor of children's performance on the Successor Task was age: An ANCOVA showed that Age was a significant predictor of Successor Task performance, $F(1, 96) = 52.23$, $p < .001$, but that children's infinity status was not, $F < 1$, *ns,* a result which likely reflects the fact that children's performance on the Infinity Task remains relatively poor until they are near ceiling on the Successor Task. While this first analysis established the somewhat unsurprising result that performance on the Successor Task improves with age, our primary goal was to establish when children become able to both (1) identify successors of known numbers and (2) understand that all possible numbers have successors, and thus that the natural numbers are infinite. To test this, we focused on the Successor Task performance of Full Infinity Knowers – i.e., children who judged that there is no highest number, and that it is always possible to add 1 to any number. Table 3 shows that the average age of Full Infinity Knowers was 6.3 years. On the hypothesis that these children comprehend the successor function, they should not only be Full Infinity Knowers, but they should also be able to infer the value of successors for most or all numbers in their productive count list. Figure 3 (Right panel) shows that although Full Infinity Knowers range widely in age, a majority performed near ceiling on the Successor Task. We also found that Full Infinity Knowers performed better on the Successor Task (M = 91.5%, SD = 14.7%) than the combined group formed by Partial Knowers and Infinity Non-Knowers (M = 81.0%, SD = 19.2%, N = 76; $t(50.17) = -2.83$, p = .0068, d = 0.66 [0.19, 1.14]).

We next asked how counting experience was related to Full Infinity Knowers' knowledge of the successor function. In a previous analysis, we showed that Very High Counters were more likely to identify particular successor relations for all numbers in their count list than those whose highest count was lower. We therefore expected that most Full Infinity Knowers should also be highly competent counters. Consistent with this, Figure 3 (Left Panel) shows that among the Full Infinity Knowers, slightly more than half were Very High Counters (14/24). Indeed, Full Infinity Knowers' median highest count was 100 (*Mean* = 75.2), suggesting that experience counting to large numbers may contribute to learning the successor function. Still, some children exhibited more modest counting ability, suggesting that experience with counting to high numbers may not be strictly necessary.

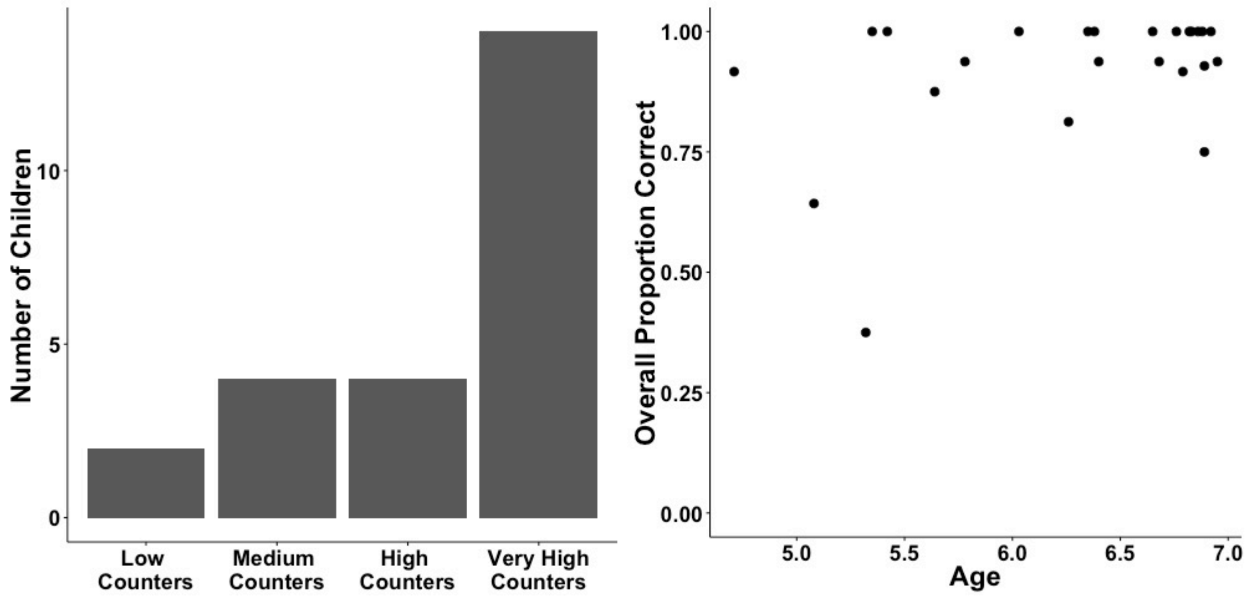

Figure 3: Among Full Infinity Knowers, the number of children in each counting group (Left Panel) and proportion correct on the Successor Task by age (Right Panel).

General Discussion

We examined children's acquisition of the successor function by asking (1) when they become able to identify the successors of familiar numbers and (2) when they know that every number *n* has a successor *n*+1, and thus that numbers never end. According to some previous studies, children have knowledge of the successor function early in childhood – either innately (Leslie, Gallistel, & Gelman, 2007) or when they become Cardinal Principle knowers (CP-knowers) at around 3½ to 4 years of age (Carey, 2004, 2009; Piantadosi, Tenenbaum, & Goodman, 2012; Sarnecka & Carey, 2008). Other studies have suggested that this knowledge emerges sometime after children learn the cardinal principle (Davidson, Eng, & Barner, 2012), providing evidence that children only have item-based knowledge of the successor function when they become CP-knowers (see also Wagner, et al., 2015). Critically, no previous study has identified when children acquire the successor function or what factors might predict its acquisition.

We addressed this by using the same measure of successor knowledge as in previous reports – i.e., the Successor Task. However, our study differed in that: (1) We tested a much wider range of numbers, (2) We tested a somewhat older group of children, and importantly, (3) We included a stronger test of knowledge of the successor function – the Infinity Task. Previous studies tested children younger than 5, and focused only on whether children understand the '+1' rule for numbers up to 30 (and sometimes much lower), making it difficult to evaluate whether they understand the successor function as a general principle that governs all possible numbers (Sarnecka & Carey, 2008; Davidson et al., 2012; Wagner et al., 2015). By testing older children with a wider range of numbers, we provide the first estimate of when children can identify the cardinal value of successors for all numbers within their counting range – i.e., at around 5½ years of age on average, or about 1½ to 2 years after they become CP-knowers. Although CP-knowers performed above chance across the range of numbers tested, their ability to identify successors was highly variable and dependent on both their age and the size of their count list. Most CP-knowers, including Low, Medium, and High Counters, performed consistently well for only the smallest numbers. In contrast, the Very High Counters consistently identified the successors for all numbers in their count lists, despite the fact that they were tested on a wider range of numbers than children in the other groups. Thus, this study establishes that children older than around 5½, and who can count up to around 80, are very likely to be able to identify the cardinal value of successors for any known number.

Nevertheless, the Successor Task alone is insufficient to probe knowledge of the successor function, as it focuses on children's understanding of particular numbers, rather than all possible numbers. As characterized by the Peano-Dedekind axioms, the successor function states that every natural number n has a unique successor n+1, and that the natural numbers are infinite. To assess the acquisition of the successor function, it is therefore important to test when children understand it as a function that governs *all possible numbers*, and not simply numbers within their count list. Clearly, this is not possible to assess using only the Successor Task. Using the Infinity Task, we asked whether children believe that every number has a successor, and also whether numbers never end. Most children (61%) held ostensibly consistent beliefs. For example, 37% (Non-Infinity Knowers) believed both that numbers have an end, and that not all numbers have a successor. Another 24% (Full Infinity Knowers) believed that numbers never end, and that all numbers have a successor. However, as in previous studies, we also found a sizeable number of children (Successor Only knowers) who believed that every number has a successor, but also that numbers must eventually end. A smaller group (Endless Only knowers) appeared to believe that numbers are infinite, but that not all numbers have a successor.

Regarding the first group – Successor Only knowers – it seems likely that children begin by learning that every number they know has a successor, and by then generalizing this to all possible numbers, before ultimately realizing that this belief implies that numbers never end – a perhaps non-obvious entailment of an unlimited successor function. The latter group is harder to square with any reasonable developmental trajectory, and as we noted in the Results, they may emerge either from random guessing, or from the fact that some parents explicitly teach their children that numbers are infinite, without necessarily teaching them about the successor function, or how to compute the successor of all numbers in their count lists. Such explicit training by parents might also explain a small number of Full Infinity knowers who would otherwise be classified as Successor Only knowers, although the other differences between these groups suggest a real developmental difference driven by age, knowledge of the count list, and better performance on the Successor Task.

Taken together, our data are consistent with a recent body of evidence which suggests that, initially, CP-knowers have limited knowledge of the logic of number words. Against the idea that becoming a CP-knower involves a semantic induction (Carey, 2004, 2009; Sarnecka & Carey, 2008), several recent studies (Davidson et al., 2012; Wagner et al., 2015; see also Le Corre, 2014) have shown that when children first become CP-knowers by Wynn's (1990, 1992) criteria, their knowledge of counting remains largely procedural, rather than reflecting an understanding of the logic of natural number. These studies suggest that the transition from subset knower to CP-knower is best described as a process of acquiring a series of increasingly sophisticated routines including (1) reciting the count list, (2) pointing at objects in one-to-one correspondence with numbers in this list, (3) learning to repeat the last number in the count in response to the question "How many?", and finally, (4) learning to run this last procedure in reverse to satisfy a request to "give" a number, by counting a set and giving all items implicated in the count. Previous studies provide strong evidence that children learn the first three procedures without yet understanding what counting means. For example, as noted by a number of previous studies (Frye, et al., 1989; Fuson, 1988; Le Corre, et al., 2006; Sarnecka & Carey, 2008), many children who are classified as subset-knowers can correctly count and report the last word in their count, but when asked to give that same number they give a random amount. Our data, together with the results from previous studies, suggest that becoming a CP-knower is yet another such procedure, and a precursor to discovering the logic of counting.

If children do not learn the successor function when they become CP-knowers, how and when might this take place? We see two broad alternatives. The first alternative is an extension of Carey's (2004, 2009) proposal. On Carey's view, the successor function is learned via a type of analogical mapping between small sets and labels for small sets. Specifically, children notice that each step up in the verbal count list corresponds to the addition of exactly one individual to a set. By noticing this analogy and then generalizing it to all possible numbers, children not only learn the successor function, but also become CP-knowers. Although this specific version of Carey's account is not likely true, since children do not appear to learn the successor function when they become CP-knowers, it nevertheless remains possible that the same mechanism applies later in development, drawing on children's ability to reliably count. Upon becoming CP-knowers, children become able to place counting sequences into correspondence with sets of objects in the world, an ability which might allow children to accurately generate sets and notice that a count of 6 differs from a count of 7 by exactly 1, and that this is also true for 7 and 8, 9 and 10, etc. (much like they later learn that 6+2=8, and that 7+2=9).

The second alternative parts from Carey's hypothesis, and is inspired by well-known facts regarding children's early struggles with learning addition and subtraction in kindergarten and first grade. At around the same age that our study finds children are mastering the successor function, previous studies have found that children are learning an almost identical skill, sometimes called "counting on" (Fuson, 1982; Fuson & Hall, 1982; Groen & Resnick, 1977; Secada, Fuson, & Hall, 1983). When children first learn to count and are shown two sets (e.g., 3 and 2), they count all objects when asked how many there are, and do this even if they are first told, e.g., that one set contains 3 items. As noted by Secada et al. (1983), training children to "count on" from a given number (in this example, 3) is generally very difficult, and can take multiple sessions over several weeks. In fact, the technique which appears to work best involves covering the labeled set, such that children cannot count all – a setup almost identical to the

Sarnecka and Carey's (2003) Unit task (or what we've called the Successor Task). These facts raise the possibility that children learn the successor function not via an analogy, but instead by making a generalization over a relatively large set of highly trained math facts – e.g., by explicitly learning in school that 4+1=5, 5+1=6 and inductively inferring that this relation holds for all numbers – i.e., that the "number after" *n* (its successor) labels the sum for all equations of the form *n*+1 (Baroody, 1995).

Our study is unfortunately unable to differentiate these two alternative hypotheses. However, due to the nature of our data, it can assess the factors which might contribute to an inductive inference more generally, whether the inputs are math facts or instead analogical relations between the count list and cardinalities. First, on either view, learning to count is important. Becoming a CP-knower provides children the opportunity to place numbers in an isomorphic relation with cardinalities, which thus allows them to discover how successive numerals differ by exactly 1 – e.g., 6 = 5 + 1; 12 = 11 + 1; etc. Based on this, children who have substantial experience counting, and noticing how counting is related to changes in cardinality, may be most likely to infer a general principle that explains the semantic relations between successive numbers, whether this is couched in math facts or otherwise. Two pieces of evidence support this idea. First, we found that performance on the Successor Task was related to how high children could count. Only the Very High counters – a majority of whom could count to 100 – could consistently identify the successors of all numbers in their count lists (see also Davidson et al., 2012). Second, Fuson et al. (1982) found that pre-kindergarten children were better at answering questions about the count sequence (e.g., What comes after seven?) than questions about cardinal relations (e.g., What is one bigger than seven?), suggesting that knowledge of the count list structure precedes learning about the relations between cardinalities (see also Fuson, 1988).

Overall, these findings are consistent with the view that mastery of counting procedures is fundamental to the acquisition of number concepts including the successor function (Carey, 2009; Sarnecka & Carey, 2008), and also addition, subtraction, and numerical ordering (Gelman & Gallistel, 1978; Gelman, 2006; Zur & Gelman, 2004). While this alone is interesting, it remains an open question *why* higher counters might perform better. One possibility is that a child's ability to count to a large number is merely a proxy for global differences in number knowledge, and is not causally implicated in their learning of the successor function. However, another possibility is that children gain insights from the count list itself. One way in which this might occur, for example, is if children who count higher become more likely to notice the recursive structure of the count list, and thus that *symbols* for larger numbers can be generated indefinitely. In support of this possibility, previous studies have shown that children who can count beyond 100 can reliably produce the within-decade structure of one through nine, and to some extent, can also correctly produce the order of decade terms, whereas those whose highest count is lower store the count sequence as a string and lack the ability to decompose it into smaller units (Siegler & Robinson, 1982; Fuson et al., 1982; see also Rule, Dechter, & Tenenbaum, 2015; Spelke, 2000; Spelke & Tsivkin, 2001). Learning such a generative system may lead children to conclude that the count list is in principle unbounded, and thus that rules which govern known numbers extend indefinitely – to all possible numbers.

Conclusion

This study is the first to document the complete developmental trajectory of successor function acquisition, and finds that it is highly protracted. Using a stronger test of successor knowledge, we found that not until after age 6 can children both identify the values of successors within their count list *and* reason about the successor function for all possible numbers. Together, these results are consistent with a recent body of evidence which suggests that acquiring the cardinal principle is just one step toward understanding natural number, rather than the end point of this process.

Acknowledgements

We would like to thank Nadia Gordon, Karen Kriger, and Eileen Wu for their assistance with data collection. Members of the Language and Development Lab provided helpful comments. We are grateful to the Reuben Fleet Science Center, day care centers, and the participating children and their families in the San Diego area. This work was supported by an NSERC CGS and MSFSS to PC and by a grant from the James S. McDonnell Foundation to DB.

References

Almoammer, A., Sullivan, J., Donlan, C., Marušič, F., Žaucer, O'Donnell, T, & Barner, D. (2013). Grammatical morphology as a source of early number word meanings. *PNAS, 110*(46), 18448-18453.

Barner, D., Chow, K., & Yang, S. (2009). Finding one's meaning: A test of the relation between quantifiers and integers in language development. *Cognitive Psychology, 58*, 195-21

Barner, D., Libenson, A., Cheung, P., & Takasaki, M. (2009). Cross-linguistic relations between quantifiers and numerals in language acquisition: Evidence from Japanese. *Journal of Experimental Child Psychology, 103*, 421-440.

Baroody, A. (1995). The role of the number-after rule in the invention of computational shortcuts. Cognition and Instruction, 189-219.

Bates, D, Maechler M, Bolker B and Walker S (2015). *lme4: Linear mixed-effects models using Eigen and S4*. R package version 1.1-9, https://CRAN.R-project.org/package=lme4.

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology, 20*, 607–618.

Brooks, N., Audet, J., & Barner, D. (2013). Pragmatic inference, not semantic competence, guides 3-year-olds' interpretation of unknown number words. *Developmental Psychology, 49*, 1066-1075.

Carey, S. (2004). Bootstrapping and the origin of concepts. *Daedalus*, 59-68.

Carey, S. (2009). *The origin of concepts*. New York: Oxford University Press.

Condry, K. F., & Spelke, E. S. (2008). The development of language and abstract concepts: The case of natural number. *Journal of Experimental Psychology: General, 137*, 22–38

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*, 162-173.

Evans, D.S. (1983). Understanding zero and infinity in the early school years (Unpublished doctoral dissertation). University of Pennsylvania.

Frege, G. (1884/1980). *The Foundations of Arithmetic: A logico-mathematical enquiry into the concept of number*. Northwestern University Press.

Frye, D., Briasby, N., Lowe, J., Maroudas, D., & Nicholls, J. (1989). Young children's understanding of counting and cardinality. *Child Development, 60*, 1158-1171.

Fuson, K. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag

Fuson, K., Richards, J., & Briars, D. (1982). The acquisition and elaboration of the number word sequence. In C. Brainerd (Ed.), *Progress in cognitive development research Vol. 1: Children's logical and mathematical cognition* (pp. 33-92). New York: Springer Verlag.

Fuson, K., & Hall, J.W (1983). The acquisition of early number word meanings: A conceptual analysis and review. In H. Ginsburg (Ed.), *The development of children's mathematical thinking* (pp. 49-107). New York: Academic Press.

Fuson, K., Pergament, G.G., Lyons, B.G., & Hall, J.W. (1985). Children's conformity to the cardinality rule as a function of set size and counting accuracy. *Child Development, 56*, 1429-1436.

Gelman, R. (1980). What young children know about numbers. *Educational Psychologist, 15*, 54-68.

Gelman, R. (2006). Young natural-number arithmeticians. *Current Directions in Psychological Science, 4*, 193-197.

Gelman, R., & Gallistel, C.R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Hartnett, P. (1991). The development of mathematical insight: from one, two, three to infinity. (Unpublished doctoral dissertation). University of Pennsylvania.

Hartnett, P., & Gelman, R. (1998). Early understandings of numbers: Paths or barriers to the construction of new understandings? *Learning and Instruction, 8*, 341-374.

Hurford, J. (1987). *Language and number: The emergence of a cognitive system*. New York: Blackwell.

Hughes, M. (1981). Can preschool children add and subtract? *Educational Psychology*, 1, pp. 207–219.

Izard, V., Streri, A., & Spelke, E. (2014). Toward exact number: Young children use one-to-one correspondence to measure set identity but not numerical equality. *Cognitive Psychology, 72*, 27-53.

Landis, J.R., & Koch, G.G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*, 159-174.

Le Corre, M. (2014). Children acquire the later-greater principle after the cardinal principle. *British Journal of Developmental Psychology, 32*, 163-177.

Le Corre, M. (2015). One-to-one correspondence principle and the cardinal principle. Poster presented at the biennial meeting of Society for Research in Child Development.

Le Corre, M., Van de Walle, G., Brannon, E., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130-169.

Le Corre, M., Li, P., Huang, B., Jia, G., & Carey, S. (In press). Numerical morphology supports early number word learning: Evidence from a comparison of young Mandarin and English learners. *Cognitive Psychology*.

Leslie, A. M., Gallistel, C.R., Gelman, R. (2007). Where integers come from. In Carruthers, P., Laurence, S., Stich, S., (Eds.) *The innate mind, Vol 3: Foundations the future*, New York: Oxford University Press, 109-138.

Lipton, J.S., & Spelke, E.S. (2006). Preschool children master the logic of number word meanings. *Cognition*, 1-10.

Mill, J. S. (1843). *A system of logic, ratiocinative and inductive* (8th ed.). New York: Harper & Brothers.

Piantadosi, S., Jara-Ettinger, J., & Gibson, E. (2014). Children's learning of number words in an indigenous farming-foraging group. *Developmental Science, 17*, 553-563.

Piantadosi, S., Tenenbaum, J., & Goodman, N. (2012). Bootstrapping in a language of thought: A formal model of numerical concept learning. *Cognition, 123*, 199-217.

R Development Core team (2013). R: A language and environment for statistical computing.

Rips, L., Asmuth, J., & Bloomfield, A. (2006). Giving the boot to the bootstrap: How now to learn the natural numbers. *Cognition, 101*, B51-B60.

Rips, L., Bloomfield, A., Asmuth, J. (2008). From numerical concepts to concepts of number. *Behavioural and Brain Sciences, 31*, 623-642.

Rule, J., Dechter, E., & Tenenbaum, J. (2015). Representing and learning a large system of number concepts with latent predicate networks. In Noelle, D. C., Dale, R., Warlaumont, A. S., Yoshimi, J., Matlock, T., Jennings, C. D., & Maglio, P. P. (Eds.) *Proceedings of* *the 37th Annual Meeting of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Sarnecka, B.W., & Gelman, S.A. (2004). Six does not just mean a lot: Preschoolers see number words as specific. *Cognition, 92,* 329-352.

Sarnecka, B.W. & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674.

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge in early childhood. *Journal of Experimental Child Psychology, 103,* 325-337.

Sarnecka, B.W., Kamenskaya, V.G., Yamana, Y., Ogura, T., & Yudovina, J.B. From grammatical number to exact numbers: Early meanings of "one," "two," and "three" in English, Russian, and Japanese. *Cognitive Psychology, 55,* 136-168.

Sarnecka, B. W., & Wright, C. E. (2013). The idea of an exact number: Children's understanding of cardinality and equinumerosity. *Cognitive Science. 37*, 1493-1506.

Sarnecka, B.W. (2015). Learning to represent exact numbers. *Synthese*, 1-18.

Schaeffer, B., Eggleston , V. H., & Scott , J. L. (1974). Number development in young children. *Cognitive Psychology, 6*, 357-379.

Secada, W.G., Fuson, K., & Hall, J.W. (1983). The transition from counting-all to counting-on in addition. *Journal for Research in Mathematics Education, 14*, 47-57.

Siegler, R., & Robinson, M. (1982). The development of numerical understandings. In H. W. Reese & L.P. Lipsett (Eds.), *Advances in child development and behavior: Vol 16* (pp. 242-312).

Spelke, E. S. (2000). Core knowledge. *American Psychologist, 55*, 1233-1243.

Spelke, E. S., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition, 78*, 45-88.

Wagner, K., Kimura, K., Cheung, P., & Barner, D. (2015). Why is number word learning hard? Evidence from bilingual learns. *Cognitive Psychology*, *83*, 1-76.

Wheeler, M. W., & Martin, W. G. (1988). Explicit knowledge of infinity. In M. J. Behr, C.B. Lacampagne, & M. W. Wheeler (Eds.), *Proceedings of the 10th Annual Meeting of the North American Branch of the International Group for the Psychology of Mathematics Education* (pp. 312-318). DeKalb: Northerin Illinois University.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*, 155-193.

Wynn, K. (1992). Child's acquisition of the number words and the counting system. *Cognitive Psychology, 24*, 220-251.

Zur, O.,& Gelman, R. (2004). Young children can add and subtract by predicting and checking. Early Childhood Quarterly Review, 19, 121-137.

Appendix A

Coding scheme

Infinity Non-Knowers included children who answered that the biggest number that they provided was the largest number (Questions 1 and 2) or that there was an end to numbers (Question 3). In cases where children responded that *X* was the biggest number possible for Question 5, this was also taken into account. Children also had to respond that it was not possible to add 1 to the biggest number (Questions 4 and 5) or for any number (Question 6).

Endless Only Knowers included children who answered the first three questions correctly (i.e., that the number that they provided was not the biggest number possible and that numbers go on forever), and responded that it was not possible to add 1 to the biggest number (Questions 4 and 5) or for any number (Question 6).

Successor Only Knowers included children who answered that they had provided the largest number (Questions 1 and 2; also from Question 5, if relevant) or that there was an end to numbers (Question 3), and that it was possible to add 1 to the biggest number (Questions 4 and 5) or for any number (Question 6).

Full Infinity Knowers included children who answered that there was no biggest number (Questions 1, 2, or 5), and that it was always possible to add 1 (Questions 4 to 6).

Appendix B

Infinity questionnaire of a child classified as an Infinity Non-Knower

E = Experimenter
C = Child (Age: 4;11)

E: What is the biggest number you can think about?
C: Ninety.
E: Is that the biggest number there could ever be?
C: Infinity. That's the biggest number ever. No number is bigger than infinity. My mom told me.
E: If I keep counting, will I ever get to the end of numbers, or do numbers go on forever?
C: Numbers go on forever until my birthday.
E: Why?
C: Because my birthday is almost coming up.
E: If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn't add any more?
C: There's a number so big we can't add anymore because it's the biggest number ever.
E: So you said that the biggest number you know is infinity. Is it possible to add 1 to infinity, or is infinity the biggest number possible?
C: Yes. Infinity and 1. It's the biggest number possible because it's the highest number ever.
E: Could I keep adding 1?
C: No
E: Why?
C: Because there's the biggest number.

Infinity questionnaire of a child classified as an Endless Only Knower

E = Experimenter
C = Child (Age: 5;3)

E: What is the biggest number you can think about?
C: A hundred and ten.
E: Is that the biggest number there could ever be?
C: No.
E: Can you think of a bigger number?
C: A million.
E: Is that the biggest number there could ever be?
C: No.
E: Can you think of a bigger number?
C: I don't know.
E: If I keep counting, will I ever get to the end of numbers, or do numbers go on forever?
C: Numbers go on forever.
E: Why?
C: Because God made them.

E: If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn't add any more?
C: You can always add to it.
E: So you said that the biggest number you know is a million. Is it possible to add 1 to a million, or is a million the biggest number possible?
C: It's the biggest number.
E: Could I keep adding 1?
C: No
E: Why?
C: (No response)

Infinity questionnaire of a child classified as a Successor Only Knower

E = Experimenter
C = Child (Age: 5;11)

E: What is the biggest number you can think about?
C: Two hundred.
E: Is that the biggest number there could ever be?
C: No
E: Can you think of a bigger number?
C: One thousand.
E: Is that the biggest number there could ever be?
C: No
E: Can you think of a bigger number?
C: Eight thousand.
E: Is that the biggest number there could ever be?
C: Yes
E: If I keep counting, will I ever get to the end of numbers, or do numbers go on forever?
C: You'll get to the end of numbers.
E: Why?
C: Numbers do not go on forever because if you keep on counting, it takes you back to 0.
E: If we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn't add any more?
C: So big a number. The biggest number in the world. Two thousand eighty-three.
E: OK! So you said that the biggest number you know is 2083. Is it possible to add 1 to 2083, or is 2083 the biggest number possible?
C: You can add 1 to 2083. It's the biggest number possible.
E: Why?
C: Because it's at the thousands.
E: Could I keep adding 1?
C: Yes.
E: Why?
C: If you keep on adding 1s, it makes another number. It takes all the way to eight thousand thirty-three.

Infinity questionnaire of a child classified as a Full Infinity Knower

E = Experimenter
C = Child (Age: 5;11)

E: What is the biggest number you can think about?
C: One hundred ten.
E: Is that the biggest number that could ever be?
C: No
E: Can you think of a bigger number?
C: A million.
E: Is that the biggest number that could ever be?
C: I can't think of any higher up numbers.
E: That's okay! Alright, so if I keep counting, will I ever get to the end of numbers, or do numbers go on forever?
C: Numbers never end I think.
E: Why?
C: Because I guess I was counting and counting one time, but I couldn't find the stop, and I went to a million, and it took me a while, and I didn't want to go any more, so I knew they went on forever.
E: So if we thought of a really big number, could we always add to it and make it even bigger, or is there a number so big we couldn't add any more?
C: No, we can add to it.
E: Why is that?
C: Because there would be a million and then you could keep adding numbers if you know them.
E: So you said that the biggest number you know is a million. Is it possible to add 1 to a million, or is a million the biggest number possible?
C: You can add 1 to it.
E: Why?
C: Because it would be like a million one, a million two.
E: Could I keep adding 1?
C: [child nodded]
E: Why?
C: Because, um, because if you want to you can keep adding and adding and make it fun.
E: So what would happen if I kept on adding one?
C: You would get really tired and just give out.
E: Alright, you did great!